Portfolio theory

Lecture 3

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Section 1

Subject of portfolio theory

Subject of portfolio theory

Model outputs

You get all the variances, you get all the co-variances, you get all the expected values: What is the best portfolio?

The question was first analysed by Markowitz (1952).

Subject of portfolio theory

Assumptions to Markowitz' Portfolio Theory

1. Investors consider each investment alternative as being represented by a probability distribution of returns.
2. Investors estimate risk on the basis of the variance of returns.
3. Investors base decisions solely on expected return and risk.
4. Investors maximize one-period expected utility.
5. Investors are risk are risk averse (diminishing marginal utility of wealth).

Section 2

Efficient risky portfolios

Efficient risky portfolios

Definitions

Definition (efficient portfolio)

Portfolio that offers the maximum expected return at a given level of risk.

Efficient risky portfolios

Definitions

Definition (μ-σ-dominance)

Portfolio A dominates portfolio B if either...

- $\mu_A \leq \mu_B$ and $\sigma_A < \sigma_B$ or
- $\mu_A > \mu_B$ and $\sigma_A \leq \sigma_B$

Efficient risky portfolios

Definitions

Definition (Efficient frontier)

locations of non-dominated portfolios in the μ-σ-space.

No rational investor would choose a portfolio that is not located on the efficient frontier.

Efficient risky portfolios

Determination of efficient portfolios

Determination of efficient portfolios of *risky* assets.

$$\sigma_P^2 = \vec{x}^T \cdot C \cdot \vec{x} \rightarrow \min_{x_0, x_1, \ldots, x_n}!$$

subject to

$$\begin{aligned} \vec{x}^T \cdot \vec{\mu} &= \bar{\mu}_P \\ \sum_{i=0}^{n} x_i &= 1 \end{aligned}$$

Solve this optimization problem with the *method of Lagrange multipliers*.

Efficient risky portfolios

Determination of efficient portfolios

Efficient frontier

$$\sigma_P(\mu_P) = \sqrt{\frac{c}{d}\left(\mu_P - \frac{a}{c}\right)^2 + \frac{1}{c}}$$

with

$$\begin{aligned} a &= \vec{\mu}^T \cdot C^{-1} \cdot \vec{1} \\ b &= \vec{\mu}^T \cdot C^{-1} \cdot \vec{\mu} \\ c &= \vec{1}^T \cdot C^{-1} \cdot \vec{1} \\ d &= b \cdot c - a^2 \end{aligned}$$

Efficient risky portfolios

Determination of efficient portfolios

Efficient frontier – minimum risk portfolio

$$\frac{\partial \sigma_P(\mu_P)}{\partial \mu_P} = 0 \quad \Leftrightarrow \mu_P = \frac{a}{c}$$

$$\Rightarrow \sigma_P = \sqrt{\frac{1}{c}}$$

Efficient risky portfolios

Conclusions

Conclusions

1. If you have just risky assets available: There is no optimal portfolio of risky assets.
2. You can eliminate risk by combining assets with negatively correlated returns.
3. There are inefficient portfolios.

Section 3

Tobin separation

Tobin separation

In so far we have just looked at risky assets.

Now we take a riskless assets into consideration.

Tobin separation

Tobin separation

1. *Diversification*: The investor always chooses the tangency portfolio of risky assets. Hence the choice of portfolio of risky assets is independent from the investor's μ-σ-preference.
2. *Leverage*: The mix between tangency portfolio and riskless asset is dependent from the investor's μ-σ-preference.