

Arbitrage Pricing Theory

Lecture 7

Dr. Martin Ewers

May 1, 2014



Table of contents

1. Arbitrage and market equilibrium
2. Multi-factor models
3. Arbitrage Pricing Theory
 - 3.1 Assumptions
 - 3.2 Arbitrage opportunities
 - 3.3 Asset return
4. APT versus CAPM

Section 1

Arbitrage and market equilibrium

Arbitrage and market equilibrium

Definition (Arbitrage)

Arbitrage is the opportunity to make a *riskless* profit from simultaneous purchases and sales.

Arbitrage and market equilibrium

Definition (Capital market equilibrium)

In capital market equilibrium, nobody wants to buy and nobody wants to sell.

Arbitrage and market equilibrium

In capital market equilibrium, there are no opportunities for arbitrage.

If there are opportunities for arbitrage, somebody will want to buy and sell.

Arbitrage and market equilibrium

If there are no opportunities for arbitrage, the capital market is not necessarily in equilibrium.

People might want to buy and sell for other reasons than opportunities for arbitrage.

Section 2

Multi-factor models

Multi-factor models

Assumption: The return of asset i is determined by factors F_1, F_2, \dots, F_n .

$$R_i = a_i + \sum_{j=1}^n b_{ij} \cdot F_j + \varepsilon_i$$

Multi-factor models

Notation

a_i abnormal return of asset i

F_j systematic factor

b_{ij} sensitivity of asset i to factor F_j

ε_i idiosyncratic component of return on asset i

By definition, idiosyncratic shocks are assumed to be uncorrelated across assets and uncorrelated with the factors.

Section 3

Arbitrage Pricing Theory

Arbitrage Pricing Theory

Assumptions

The APT is based on less assumptions than the CAPM.

Assumption		Comment
1.	All securities have finite expected values and variances.	
2.	Some (i.e. not necessarily all) agents can form zero-risk portfolios.	CAPM: All (!) investors hold the same portfolio.

Arbitrage Pricing Theory

Assumptions

The APT is based on less assumptions than the CAPM.

	Assumption	Comment
3.	No taxes.	
4.	No transaction costs.	
5.	Absence of arbitrage.	In contrast to the CAPM, market equilibrium is not necessarily required.

Arbitrage Pricing Theory

Assumptions

The APT is based on less assumptions than the CAPM.

	Assumption	Comment
6.	Linear relationship between expected return of assets and factor loadings.	APT is a multi-factor or a single factor model. The SIM is always a single factor model. The CAPM is not a factor model.

Arbitrage Pricing Theory

Arbitrage opportunities

Definition (Arbitrage portfolio)

Criterion 1: The portfolio is **self-financing**, i.e. the sum of shares in assets is zero.

$$\sum_{i=1}^n x_i = 0$$

Arbitrage Pricing Theory

Arbitrage opportunities

Definition (Arbitrage portfolio)

Criterion 2: The portfolio has **no systematic risk**, i.e. the portfolio's sensitivities with regard to all factors is zero.

$$b_{P_k} = 0$$

$$\text{with } b_{P_k} = \sum_{i=1}^n x_i \cdot b_{ik}$$

Arbitrage Pricing Theory

Arbitrage opportunities

Definition (Arbitrage portfolio)

Criterion 3: The portfolio has **no idiosyncratic risk**.

$$\varepsilon_P = 0$$

with
$$\sum_{i=1}^n x_i \cdot \varepsilon_i = 0$$

Arbitrage Pricing Theory

Arbitrage opportunities

Criteria for absence of arbitrage

- ▶ The expected return of any arbitrage portfolio is zero:

$$E[R_P] = 0$$

- ▶ If the pay-off from a security is (i) positive / (ii) negative / (iii) zero in all states of the environment, the security's price has to be (i) positive / (ii) negative / (iii) zero.
- ▶ Two securities that always have the same payoff must have the same price.

Arbitrage Pricing Theory

APT as a factor model

Assumption: The future return of asset i is determined by factors F_1, F_2, \dots, F_n .

$$\begin{aligned} R_i &= a_i + \sum_{j=1}^n b_{ij} \cdot F_j + \varepsilon_i \\ &= r_0 + \sum_{j=1}^n b_{ij} \cdot (R_{Fj} - r_0) + \varepsilon_i \end{aligned}$$

Arbitrage Pricing Theory

APT as a factor model

Interpretation of a_i

If there is a risk-free asset, the return of portfolios with no risk (with all b 's equal to zero) will equal the risk-free rate:

$$a_i = r_0$$

Arbitrage Pricing Theory

APT as a factor model

Expected return of an asset i

$$E[R_i] = r_0 + \sum_{j=1}^n b_{ij} \cdot (E[R_{Fj}] - r_0)$$

where

r_0 risk-free rate

$E[R_{Fj}] - r_0$ risk premium of the factor j

Section 4

APT versus CAPM

APT versus CAPM

Similarities APT — CAPM

- ▶ Expected returns depend on systematic (non-diversifiable) risk.
- ▶ Expected returns do not depend on idiosyncratic (asset-specific) risk.

APT versus CAPM

Differences APT — CAPM

- ▶ APT does not require investors to hold any particular portfolio. There is no special role for any market portfolio.
- ▶ APT requires just one investor who eliminates arbitrage opportunities.
- ▶ APT acknowledges that there may be several non-diversifiable risk factors.